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A Static Plane Symmetric Cosmological Model of the Universe with Logotropic Equation of State

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Abstract

We propose a Static Plane Symmetric cosmological model using a single "dark fluid" with a logotropic equation of state $P = A\ln(\frac{\rho}{\rho_p})$, *where* ρ *is the rest-mass density, the Planck density* $\rho_p = 5.16x10^{99}\,gm^{-3}$ *and* A *is the logotropic temperature. We also discuss the physical behaviour of the solutions by using some physical parameters.*

Keywords : *static plane symmetric space-time, dark fluid, logotropic temperature.*

Introduction

The universe has been described to be isotropic and homogeneous by the standard Friedmann-Robertson-Walker cosmological model. At the early stages of evolution, many cosmological models have been investigated to describe the nature of the universe. Bianchi space times have been widely used to describe homogeneous cosmological models. Many authors have attempted to propose homogeneous and anisotropic models of the universe. Raj Bali et al[2] have studies Bianchi III cosmological model with variable G and Λ. Bianchi type V models have been studied with bulk viscous matter and time varying gravitational constant by Baghel& Singh[1]. Chandel et al[4] have described Bianchi type VI_0 dark energy cosmological model. AnirudhPradhan[11]has discussed accelerating dark energy models with anisotropic fluid in Bianchi type VI_0 space time.

The nature of dark matter and dark energy is still unknown. This has been a matter of interest for many researchers. Dark energy (DE) has been introducedin cosmology to account for the acceleration of the expansion of the universe. In the standard cold dark matter (ΛCDM) model[7], dark matter (DM) is represented by a pressureless fluid and DE is considered by cosmological constant Λ introduced by Einstein. The ΛCDM encounters few problems at the galactic scale but works quite well at cosmological scale[10]. It predicts that DM halos should be cuspy, [9] while according to observations, they appear to have a flat core[3].

These are referred to as the"cusp problem" and "missing satellite problem". For solving the small-scale crisis of ΛCDM, some authors have proposed that quantum pressure prevents gravitational collapse and leads to cores instead of cusps.

On the other hand, at the cosmological scale, the ΛCDM model faces two problems. The first one is the cosmological coincidence, that is, why is the ratio of DE and DM of order unity if they are two different entities. The second one is the cosmological constant problem. We know that the cosmological constant is equivalent to constant energy density $\varepsilon_\Lambda = \rho_\Lambda c^2$ with an equation of state $P = -\varepsilon$. Cosmological observations lead to the value of $\rho_\Lambda = \frac{\Lambda}{8\Pi G} = 6.72x10^{-24}\,gm^{-3}$ of cosmological density DE. However, according to particle physics and quantum field theory, the vacuum energy which is interpreted as cosmological constant should be of the order of Planck density $\rho_p = 5.16x10^{99}\,gm^{-3}$. The ratio between Planck density ρ_p and cosmological density ρ_Λ is $\frac{\rho_p}{\rho_\Lambda} \approx 10^{123}$. This is where the cosmological problem starts from.

To counteract this problem, some authors have proposed to abandon the cosmological constant and to explain the acceleration of the universe in terms of DE with a time-varying density called "quintessence"[12]. Further Kamenshchik et al[8] have proposed a heuristic unification of DM and DE with an equation of state $P = -A/\varepsilon$ called the Chaplygingas. This uses DM as a pressureless fluid at early times and a fluid with constant energy density DE at late times. But this model does not agree with the observational data. Hence, this has motivated us to study a cosmological model of Static Plane Symmetric space-time with logotropic equation of state $P = A\ln(\frac{\rho}{\rho_p})$ [5]. We propose a new cosmological model of the universe with the unification of DM and DE as a single "dark fluid" DF [6]. It is calledlogotropic fluid LDF.

A] Logotropic Cosmology

The Friedmann equations for a flat universe without cosmological constant are[13] :

$$\frac{d\varepsilon}{dt} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0,$$
$$H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\Pi G}{3c^2}\varepsilon \qquad (1)$$

Where ε (t) is the energy density, P(t) is the pressure and a(t) is the scale factor and H is the Hubble parameter.

For a relativistic fluid at T=0, the first law of thermodynamics reduces to[14] :

$$d\varepsilon = \frac{P+\varepsilon}{\rho}d\rho \qquad (2)$$

Where ρ is the rest-mass density.

Combined with the equation of continuity we get

$$\frac{d\rho}{dt} + 3\frac{\dot{a}}{a}\rho = 0 \qquad (3)$$

We have used an equation of state which is of the form P=P(ρ), then equation (2) can be integrated to obtain the relation between energy density and rest-mass density as :

$$\varepsilon = \rho c^2 + u(\rho) \qquad (4)$$

We observe that energy density ε is obtained as a sum of rest-mass density ρc^2 and internal energy $u(\rho)$. The rest-mass density is positive whereas the internal energy can be positive or negative. But the total energy $\varepsilon = \rho c^2 + u(\rho)$ is always positive.

We assume that the universe is filled with a single dark fluid DF described by the logotropicEOS :

$$P = A\ln(\frac{\rho}{\rho_*}) \qquad (5)$$

There are two unknown parameters in this model : A (logotropic temperature) and reference mass density ρ_*.

Using equation (4) and (5) and assuming $\rho_* = \rho_p$, we get the relation between energy density and rest-mass density as :

$$\varepsilon = \rho c^2 - A\ln(\frac{\rho}{\rho_p}) - A = \rho c^2 + u(\rho) \qquad (6)$$

Therefore, the term ρc^2 refers to DM and the internal energy term $u(\rho)$ refers to DE. This leads to a natural unification of DM and DE and elucidates their mysterious nature.

B] The Static Plane Symmetric Space-time is as follows :

$$ds^2 = e^{2\alpha}dt^2 - dx^2 - e^{2\beta}(dy^2 + dz^2) \qquad (7)$$

Where α, β are functions of x.

The field equations are given by :

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \qquad (8)$$

Where R_{ij} is Ricci Tensor, R is Ricci Scalar and T_{ij} is energy momentum tensor.

The energy momentum tensor T_{ij} for single Dark Fluid (DF) is given by :

$$T_{ij} = (p+\rho)u_i u_j - pg_{ij} \qquad (9)$$

Where ρ is rest-mass density, p is the pressure and u^i is the four velocity vector satisfying $g_{ij}u^i u_j = 1$

C] The Field Equations :

Using equations (7), (8) and (9) the field equations are as follows :

$$\ddot{\beta}+2\dot{\beta}^2+\ddot{\alpha}+\dot{\alpha}^2+\dot{\alpha}\dot{\beta}=-p \qquad (9)$$

$$\ddot{\beta}+2\dot{\beta}^2+\ddot{\alpha}+\dot{\alpha}^2+\dot{\alpha}\dot{\beta}=-p \qquad (10)$$

$$2\ddot{\beta}+3\dot{\beta}^2=-p \qquad (11)$$

$$\ddot{\beta}+2\dot{\alpha}\dot{\beta}=\rho \qquad (12)$$

We have four unknown variables and three equations so we introduce the relation :

$$\alpha-2\beta=0 \qquad (13)$$

Subtracting equation (10) from (11) and using equation (13) we get :

$$\ddot{\beta}+4\dot{\beta}^2=0 \qquad (14)$$

This is a non-linear differential equation. By using substitution method and from equation (13) we get the solution as :

$$\beta=\frac{1}{4}\ln(ax+k) \qquad (15)$$

$$\alpha=\frac{1}{2}\ln(ax+k) \qquad (16)$$

Where a is an arbitrary constant and k is constant of integration.

Therefore, we can express the static plane symmetric space-time as :

$$ds^2=(ax+k)dt^2-dx^2-(ax+k)^{1/2}(dy^2+dz^2) \qquad (17)$$

Now considering equation (12), (13), (15) and (16) we get rest-mass density as :

$$\rho=\frac{5a^2}{16(ax+k)^2} \qquad (18)$$

Taking suitable values of a=1 and k=0 we get :

$$\rho=\frac{5}{16x^2} \qquad (19)$$

From equation (5) and (19), we can get the value of pressure as :

$$P=A\ln\left(\frac{5}{16\rho_p x^2}\right) \qquad (20)$$

Total energy density is therefore given by using equation (6), (19) and (20) as :

$$\varepsilon=\frac{5c^2}{16x^2}-A\ln\left(\frac{5}{16\rho_p x^2}\right)-A \qquad (21)$$

Using equation (20) and (21), the energy density parameter which is given by $\omega=P/\varepsilon$ can be written as :

$$\omega=\frac{A\ln\left(\frac{5}{16\rho_p x^2}\right)}{\frac{5c^2}{16x^2}-A\ln\left(\frac{5}{16\rho_p x^2}\right)-A} \qquad (22)$$

Conclusion :

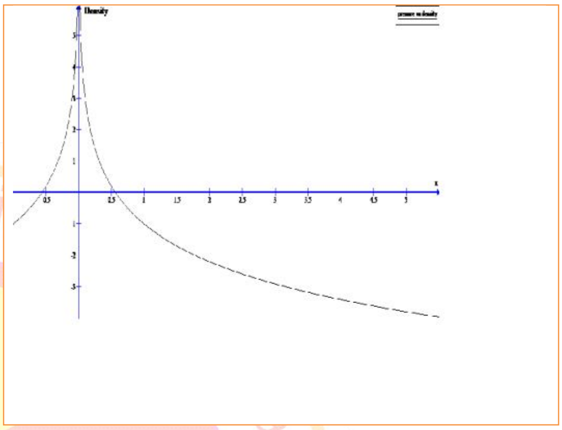

Fig. 1 Pressurevs Density

In this model, we have not used cosmological constant, dark matter and dark energy. Therefore the cosmological constant problem and cosmological coincidence problem has been taken care of. All properties of dark matter and dark energy can be related to a single dark fluid. DM relates to rest-mass density and DE mimics internal energy. The logotropic constant A can be interpreted as a fundamental constant of physics superseding the cosmological constant.

The logotropic temperature $A\geq 0$ is related to generalised thermodynamics and corresponds to log-entropy. It shows that the universe is **"isothermal"** [12]. The logotropic EOS could be a good candidate for unification of Dark matter and Dark energy. It provides a good description of the cosmological evolution of the universe and accounts for many properties of the DM halos, some of them unexplained uptil now. By using the static plane symmetric metric, we have obtained that density is independent of time from equation (18). From **Fig. 1**, we get $P<0$ as $x\to\infty$. Therefore, negative pressure indicates that the universe is expanding. This EOS is a strong incentive to study in future as a unification of DM and DE to form a single DF.

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